Proof Without Words: The Pythagorean Theorem
with Equilateral Triangles

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The Pythagorean theorem (Proposition I.47 in Euclid’s Elements) is usually illustrated with squares drawn on the sides of a right triangle. However, as a consequence of Proposition VI.31 in the Elements, any set of three similar figures may be used, such as equilateral triangles as shown at the right. Let $T$ denote the area of a right triangle with legs $a$ and $b$, and hypotenuse $c$; let $T_a$, $T_b$, and $T_c$ denote the areas of equilateral triangles drawn externally on sides $a$, $b$, and $c$; and let $P$ denote the area of a parallelogram with sides $a$ and $b$ and $30^\circ$ and $150^\circ$ angles. Then we have

**Lemma.** $T = P$.

*Proof.*

\[ T_a + 2T = T_a + 2P \implies T = P. \]

**Theorem.** $T_c = T_a + T_b$.

*Proof.*

\[ T_c + 3T = T_a + T_b + 3P \implies T_c = T_a + T_b. \]

**Summary.** A visual proof of a modified Pythagorean theorem, showing that the area of an equilateral triangle constructed on the hypotenuse of a right triangle equals the sum of the areas of equilateral triangles constructed on the legs.

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